



A Montessori Unit for First, Second, and Third Grade Students

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<http://ouray.cudenver.edu/~eaross/portfolio/geometryunit.pdf>



This unit is designed to span a focus period using Montessori manipulatives, geometry materials from traditional educational methods, and a variety of websites and software. The unit was a recipient of the Xcel Energy Foundation Classroom Connection Teacher Network dissemination grant. It has also been taught in various seminars at the Montessori Education Center of the Rockies in Boulder, Colorado.

The standards, as per the Montessori geometry curriculum and Colorado State Standards, are color-coded by grade level. Those addressing first grade standards are in **green** type. Those addressing second grade standards are **blue**. Third grade standards are **red**. Each grade level should briefly review those standards introduced in prior grades.

In addition, this unit meets the following ISTE NETS (<http://cnets.iste.org>):

- NETS•S 1, 3
- NETS•T I, II, III
- NETS•A II

Integrated into the lessons are technology skill-building exercises, such as how to create a chart or how to use the AutoShape menu of Microsoft Word to create basic shapes. These standards will be in **purple** font.

Some of the lessons include activities that could easily be incorporated into homework. One lesson may not necessarily be given in one sitting. The order of the lessons may be changed as per teacher preference.

Rationale:

Besides for the purposes of meeting state and Montessori standards, geometry theorems are taught because it helps to give the children a good foundation for future work in physics, architecture, seeing patterns in nature, and measurement.

The use of technology helps teachers to speak to different learning styles, allows easy access for students to revisit lessons both within the classroom and at home, keeps the students' interest with the variety and sophistication of the lessons, and integrates the technology skills that the student will need in the future. The technology component also gives teachers and students information at their fingertips without the need for expensive, cumbersome, outdated textbooks. The applets have perfection in their design so that the learner does not need to worry about the manipulatives being placed incorrectly. Beyond the initial cost of having the technology available, money is saved by having multiple resources available to all students, rather than having to share materials or purchase multiple sets of very expensive materials.

Though many of the lessons say that the skills may be further explored using the software, the programs and websites listed are of excellent quality and should be used as a teaching tool with the students with the same respect as given to the Montessori manipulatives.

Also, where appropriate, the Latin or Greek root words and meanings follow geometry terms. Learning the etymology of new words aids in student comprehension of the history and meaning of the terms.

Materials Needed:

***An addendum is at the end of this unit showing pictures of the Montessori materials so that teachers may create or purchase their own sets.**

- Computers with CD-ROM, access to the Internet, Microsoft Office
- Printer
- Montessori geometric solids
- Montessori geometric cabinet
- Montessori Pythagorean material
- Montessori geometric stick box

- Montessori constructive triangles, boxes 1 – 3
- Montessori fraction cut-outs
- Montessori volume cube
- Montessori number rods or cubing material (only units needed)
- Montessori 360 degree template
- Montessori geometric nomenclature cards (*can be used for student enrichment throughout the unit, though they have not specified as materials in the lessons)
- Protractors for each child
- Rulers for each child
- Compasses for each child
- Straws, modeling clay, tape, glue, scissors, string, paper
- Montessori golden beads: one unit, one ten, one hundred, and one thousand to represent the point, line, plane, and solid
- Edmark Mighty Math series:
 - Carnival Countdown (K-2)
 - Zoo Zillions (K-2)
 - Number Heroes (3-6)
 - Calculating Crew (3-6)
 - Cosmic Geometry (7-9)
 - Mighty Math resource notebooks for teachers (these will not be addressed specifically in the lessons, but provide great supplemental material to the lessons)
- LCD projector or TV-ator
- Dry erase board, interactive whiteboard, or similar tool
- Digital camera

	Materials needed	Standards Met	Supplementary Resources
Lesson 1: Point, line, plane, solid	Golden bead unit, ten, hundred, & thousand, Square from geometric cabinet, Cube from geometric solids, dry erase board, LCD projector, computers	<ul style="list-style-type: none"> • Can identify and define point, line, plane, and solid. • Can identify dimension of an object. • Can give real-life examples of 2D and 3D objects. • Understands that point can also be called a vertex. • Can plot points on x and y axis. • Can enter a web address and save the bookmark into the correct folder. 	<ul style="list-style-type: none"> • Primary Games website
Lesson 2: Lines, line segments, rays	Montessori geometric stick box, dry erase board, computer, LCD projector *a computer and projector would be useful in all lessons	<ul style="list-style-type: none"> • Can identify and define line, line segment, ray. • Can identify parallel, intersecting, and perpendicular lines. 	<ul style="list-style-type: none"> • Number Heroes: Geoboard • Cosmic Geometry: Geo Academy • BrainPOP website
Lesson 3: Horizontal and Vertical Lines	Bowl of water, blue food coloring, stick from the geometric stick box, globe, small figure of a person, plumb line and string from the geometric stick box	<ul style="list-style-type: none"> • Can identify and define horizontal and vertical lines 	
Lesson 4: Identifying angles	Montessori geometric stick box, protractor, dry erase board	<ul style="list-style-type: none"> • Can identify and define an angle • Can identify, define, and create acute, obtuse, and right angles. • Can measure an angle using a protractor • Can create simple shapes using Draw application 	<ul style="list-style-type: none"> • Number Heroes: Geoboard • Cosmic Geometry: Geo Academy • Cosmic Geometry: Amazing Angles • AutoShapes in Word to re-create angles • BrainPOP website

	Materials needed	Standards Met	Supplementary Resources
Lesson 5: Working with angles	Protractor, compass, ruler	<ul style="list-style-type: none"> • Can copy and bisect an angle using a compass. 	
Lesson 6: Traits of angles	Montessori geometric stick box, protractor, angle study dominoes from Taskmaster	<ul style="list-style-type: none"> • Can find complementary and supplementary angles • Can find value of vertical angles, corresponding angles, and opposite interior angles. 	<ul style="list-style-type: none"> • Cosmic Geometry: Geo Academy • Cosmic Geometry: Amazing Angles • Study Works Online website
Lesson 7: Geometry cabinet nomenclature	Montessori geometric cabinet	<ul style="list-style-type: none"> • Can identify circle, oval, ellipse, square, rectangle, triangle, quatrefoil, trapezoid, common quadrilateral, curvilinear triangle, rhombus, parallelogram • Can create simple shapes using Draw application 	<ul style="list-style-type: none"> • AutoShapes in Microsoft Word for children to re-create shapes • Carnival Countdown: Pattern block roundup • Number Heroes: Geoboard • Cosmic Geometry: Geo Academy • Cosmic Geometry: Amazing Angles • Illuminations website • BrainPOP website
Lesson 8: Perimeter and area of quadrilaterals	Montessori stamp game, dry erase board	<ul style="list-style-type: none"> • Can find perimeter and area of a simple square or rectangle 	<ul style="list-style-type: none"> • Carnival Countdown: Pattern block roundup • Number Heroes: Geoboard • Cosmic Geometry: Geo Academy • Cosmic Geometry: Amazing Angles

	Materials needed	Standards Met	Supplementary Resources
Lesson 9: Symmetry	Montessori geometric shapes, string, blue constructive triangles	<ul style="list-style-type: none"> • Can identify a line of symmetry on a shape • Can identify radial symmetry versus linear symmetry 	<ul style="list-style-type: none"> • BrainPOP website • Carnival Countdown: Pattern block roundup • Number Heroes: Geoboard • Cosmic Geometry: Geo Academy
Lesson 10: Types of triangles	Montessori geometric stick box, Montessori geometric cabinet drawer with triangles, red, blue, and yellow triangles from "Detective Adjective Game," Montessori constructive triangles (2 nd box)	<ul style="list-style-type: none"> • Can classify a triangle according to angles and sides (right, acute, obtuse, equilateral, isosceles, scalene) 	<ul style="list-style-type: none"> • Illuminations Geoboard • Number Heroes: Geoboard • Cosmic Geometry: Geo Academy • Cosmic Geometry: Amazing Angles • BrainPOP website
Lesson 11: Traits of triangles	Montessori triangle drawer, Montessori geometric sticks, protractor	<ul style="list-style-type: none"> • Can determine if three sides will create a triangle or not • Can find missing angle of a triangle using the 180 rule 	<ul style="list-style-type: none"> • Cosmic Geometry: Geo Academy • Cosmic Geometry: Amazing Angles
Lesson 12: Pythagorean theorem	Montessori Pythagorean material	<ul style="list-style-type: none"> • Can give definition of the Pythagorean Theorem and find a missing side of a triangle using the theorem 	<ul style="list-style-type: none"> • Cosmic Geometry: Geo Academy • Cosmic Geometry: Amazing Angles • Study Works websites • Illuminations website • BrainPOP website

	Materials needed	Standards Met	Supplementary Resources
Lesson 13: Regular polygons	Montessori geometric cabinet, Montessori geometric sticks, blue constructive triangles, Montessori constructive triangles (1 st box), ruler, protractor	<ul style="list-style-type: none"> • Can identify and define a regular polygon • Can create polygons with other shapes • Can find the perimeter of a polygon • Can find missing angle of a polygon based on the 360 rule 	<ul style="list-style-type: none"> • Illuminations websites • Number Heroes: Geoboard • Cosmic Geometry: Geo Academy
Lesson 14: Traits of the circle, ellipse	Montessori geometric cabinet, string, flat cork board from geometric stick box, tacks, compasses for children	<ul style="list-style-type: none"> • Can identify circle and ellipse • Can identify the foci • Can find the radius, diameter, and circumference • Can give a definition of pi 	<ul style="list-style-type: none"> • BrainPOP website
Lesson 15: Further study of circles	Montessori fraction cut-outs, Montessori 360 degree template, large circle from geometric cabinet	<ul style="list-style-type: none"> • Can find degree measurement of a sector and compare it to a circle being 360 degrees • Can identify a chord, arc, and sector 	<ul style="list-style-type: none"> • Cosmic Geometry: Geo Academy • Cosmic Geometry: Amazing Angles • BrainPOP website
Lesson 16: Similar, equivalent, and congruent	Montessori stamp game, hexagon & triangle plates, red, blue, and yellow triangles from Detective Adjective Game	<ul style="list-style-type: none"> • Can classify two shapes as being similar, equivalent, or congruent 	<ul style="list-style-type: none"> • Number Heroes: Geoboard • Cosmic Geometry: Geo Academy • Cosmic Geometry: Amazing Angles

	Materials needed	Standards Met	Supplementary Resources
Lesson 17: Montessori geometric solids	Montessori geometric solids, Montessori volume cube, units from Montessori number rods or cubing material	<ul style="list-style-type: none"> • Student can identify and define ovoid, ellipsoid, cylinder, cube, sphere, square-based pyramid, triangular-based pyramid, triangular prism, rectangular prism, cone • Can find volume and surface area of simple solids • Can create simple charts in a word processing document or in a spreadsheet 	<ul style="list-style-type: none"> • Digital photos of each solid • Zoo Zillions: 3D Gallery • Calculating Crew: Dr. Gee's 3D Lab • Cosmic Geometry: Amazing Angles • BrainPOP website
Lesson 18: Exploration with complex geometric solids	Straws, modeling clay, tape or glue, paper, scissors	<ul style="list-style-type: none"> • Student can identify characteristics of complex solids 	<ul style="list-style-type: none"> • Illuminations website and grid to record information
Lesson 19: Practical application of geometry	WebQuest: "Designing and building a Montessori playground," computers with internet access, popsicle sticks, glue, cardboard, aluminum foil, clay, sand, other materials for building, PowerPoint	<ul style="list-style-type: none"> • Students will use geometry skills to serve practical purposes • Students will create simple PowerPoint presentations 	<ul style="list-style-type: none"> • Websites and booklet as provided by the WebQuest

Lesson 1: Point, Line, Plane, Solid

This lesson is to introduce the idea of the point, line, plane, and solid and their corresponding dimensions (zero, one-dimensional, 2D, and 3D), to familiarize 2nd and 3rd graders with plotting points on an x and y axis, and for children to get experience in finding websites, bookmarking them, and placing the bookmarks into a folder.

Standards Addressed:

- Can identify and define point, line, plane, and solid.
- Can identify dimension (2D, 3D, etc)
- Can give real-life examples of objects with 2 and 3 dimensions.
- Understands that point can also be called a vertex.
- Can plot points on an x and y axis.
- Can enter a web address and save the bookmark into the correct folder.

Materials and Resources needed:

- Golden bead unit, ten, hundred, and thousand
- Square from geometric cabinet
- Cube from geometric solids
- Dry erase board
- Computer with projector. Computers for students.
- Primary Games website: <http://www.primarygames.co.uk/index2.html>

Procedure:

1. Explain that this is the first lesson in a geometry (Greek: geo = world, metron = measure) unit.
2. Converse with students to see how many have been to a 3D movie. What was the difference?
3. Have students compare the flat square to the cube. Encourage them to express the differences and similarities that they see.
4. Explain that what they are talking about are dimensions: how many directions an item can go.
5. Place the unit on a rug. The bead only *represents* a point in space. Actual points have no dimensions and take up no space. These are also known as "vertices."
6. Begin lining up beads until a line is formed. Explain that lines are one-dimensional and are made up of a set of points. Students may be confused that lines seem to go in two directions. In theory, lines go on forever. Exchange the lines of beads for a ten bar.
7. Place the hundred square on the rug. Students may notice that it is relatively flat. The hundred represents a plane, or two-dimensional object, having width and height.
8. Finally, place the thousand cube on the rug. The thousand represents a solid, having three dimensions (width, height, and depth.)
9. Give examples that represent a point, line, plane, or solid and have children name the dimension. Have the children name examples that they can think of.
10. Sometimes, points are plotted on what is called a grid. Draw a very simple example of a grid with an X and Y axis, numbered to no more than 5.
11. Explain that the X axis is horizontal and the Y axis is the vertical. Points are always plotted X first, then Y. Show them an example (4, 2) and plot the point.
12. Direct the children to the website on the projector, showing them how to bookmark it, create a "Math" folder, and save the website to the folder.
12. Have the children play **Billy Bug and his Quest for Grub** for further plotting practice.

Lesson 2: Lines, line segments, and rays

This lesson is to further examine relationships between lines and point.

Standards Addressed:

- Can identify and define line, line segment, and ray.
- Can identify parallel, intersecting, and perpendicular lines.

Materials and Resources needed:

- Number Heroes: Geoboard, Levels A, H, J, R
- Cosmic Geometry: Geo Academy Levels A, C
- BrainPOP website: "Parallel and Perpendicular Lines"
<http://www.brainpop.com/math/geometry/parallelandperpendicularlines>
- Dry erase board or similar tool
- Montessori geometric stick box
- Computer and projector

Procedure:

1. Remind students of previous conversation when they discussed that lines, in theory, go on forever. In geometry, lines are represented by a small line with arrows at either end. Draw an example.
2. Lines that do not go on forever have endpoints. These are called line segments. Draw a line segment with a point on either end.
3. A line that starts at one point and goes on forever in the other direction is called a ray. Draw a ray on the board.
4. Shapes with straight edges are created using points and lines. These can be known as vertices and sides.
5. Lines that cross are known as intersecting (Latin: inter=between, sect=cut) lines. With the geometric sticks, show two lines crossing, not at perpendicular angles.
6. Lines never cross are known as parallel (Greek: para=equivalent to, alleles=one another, side by side) lines. Place two more sticks on the rug that are parallel.
7. Lines that cross to make a perfect corner, or at 90 degrees, are perpendicular (Latin: perpendiculum: plumb-line.)
8. Have the children re-create intersecting, parallel, and perpendicular lines using the geometric sticks.
9. Further explore the concepts using Number Heroes. Do a few together from the LCD projector, then allow students time to explore.
10. Students can test knowledge on Cosmic Geometry or watch BrainPOP movie.

Lesson 3: Horizontal and Vertical Lines

This lesson is to introduce the concepts of horizontal and vertical.

Standards Addressed:

- Can identify and define horizontal and vertical lines

Materials and Resources needed:

- Bowl of water
- Blue food coloring
- Stick from the geometric stick box
- Globe
- Small figure of a person
- Plumb line and string from the geometric stick box

Procedure:

1. HORIZONTAL LINES: Fill bowl with water and color blue with food coloring.
2. Place the stick in the water. Ask the children to tell what they notice. Many may say that the line is "flat." Ask them to notice the relationship of the stick to the floor. The two lines are parallel.
3. With your hands, tilt the bowl. Ask if the line is still parallel to the floor. (It is.) The stick represents a horizontal line. Horizontal lines are parallel to the Earth. Children may have seen the horizon before at the beach or on a grassy plain. Ask the children to identify other horizontal lines in the room.
4. Place and hold the figure on the globe. Have the children notice that a horizontal line to the person is actually part of a curved line of the Earth, even though it appears straight.
5. VERTICAL LINES: Next, hold the plumb line from the top of the string and let the weight hang straight down. The children may notice that it is pointed "down."
6. Next, pull the string partly from the middle so that the top half of the string is at an angle, while the bottom half still hangs straight down.
7. Place the stick on the floor and hang the plumb line so that the weight and the stick touch. Have the children notice that the two lines represented here appear to be perpendicular. These two lines are vertical. A vertical line meets the surface of the Earth at a right angle. Have the children identify other vertical lines in the room.
8. Place and hold the figure on the globe. If the figure were holding a plumb line, where would the plumb line hang? Explore the concept of "down" and have children realize that down is actually pointing to the center of the Earth.

Lesson 4: Identifying Angles

This lesson is to explore definitions, classification, and some operations with angles.

Standards Addressed:

- Can identify and define an angle
- Can identify, define, and create acute, obtuse, and right angles.
- Can measure an angle using a protractor
- Can create simple shapes using Draw application

Materials and Resources needed:

- Montessori geometric stick box
- Protractor
- dry erase board
- Number Heroes: Geoboard, Levels B, I
- Cosmic Geometry: Geo Academy, Level B
- Cosmic Geometry: Amazing Angles, Levels A, B
- AutoShapes in Word to re-create angles
- BrainPOP Website: "Measuring Angles"
<http://www.brainpop.com/math/geometry/measuringangles>

Procedure:

1. Using dry erase board, draw an angle. See if children can identify points and rays. Have them come up with a definition for an angle (Latin: angulus = ankle.)
2. Give each child two matching sticks from the geometric stick box and have them create acute (Latin: acutus=sharp), right, and obtuse (Latin: obtusus = dull) angles. Children also enjoy creating angles with their arms.
3. Second graders, who have had exposure to degrees in a circle, can begin to measure angles drawn on a sheet of paper. Having a large protractor to demonstrate helps. Children often get confused as to which set of numbers on a protractor to use. Remind them to use their knowledge of acute and obtuse angles to test to see if their answer is logical.
4. Have children explore concepts using the software. Again, do a few together first using the projector.
5. Using AutoShapes in Word, have children recreate acute, obtuse, and right angles.
6. Children may enjoy watching BrainPOP movie about measuring angles.

Lesson 5: Working with Angles

This lesson is to further the study and manipulation of angles.

Standards Addressed:

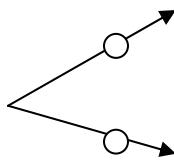
- Can copy and bisect an angle using a compass.

Materials and Resources needed:

- Protractor
- Compass
- Ruler

Procedure:

1. With a protractor, draw an angle with a number that is easily divided by two, for example, an 80 degree angle.
2. Explain that dividing the angle mathematically is one way to bisect (Latin: bi=two, sect=cut), or exactly cut in half, an angle, but is not the most precise method.
3. On a sheet of paper, draw an angle.
4. With a compass, mark a point on each of the two rays by spreading the pencil end and the point of the compass across the angle.
5. Placing the point of the compass on one of these points, make a small arc in front of the angle. Repeat by placing the point of the compass on the second point made in step 4 and make another arc. The two arcs should cross.
6. Draw a line from the original point of the angle to the point where the two arcs cross. This should bisect the angle.
7. Copying angles: draw an arc across the two rays of the angle with a compass. Keeping the compass still, draw an identical arc across the ray of the angle to become a copy.
8. With the compass, mark the distance between the two cross points on the first angle. Mark the same distance on the second angle.
9. Draw a line from the point of the second angle to the where the large arc and small mark meet.



Lesson 6: Traits of Angles

The purpose of this lesson is to familiarize students with complementary, supplementary, vertical, and opposite interior angles.

Standards Addressed:

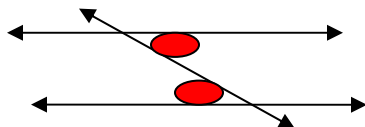
- Can find complementary and supplementary angles
- Can find value of vertical angles, corresponding angles, and opposite interior angles.

Materials and Resources Needed:

- Montessori geometric stick box
- Protractor
- Angle study dominoes from Taskmaster
- Cosmic Geometry: Geo Academy, Level B
- Cosmic Geometry: Amazing Angles, Levels B, J
- Study Works Online website:
 - (Alternate angles) http://www.studyworksonline.com/cda/content/applet/0,,NAV2-21_SAP211,00.shtml
 - (Corresponding angles) http://www.studyworksonline.com/cda/content/applet/0,,NAV2-21_SAP210,00.shtml
 - (Vertical angles) http://www.studyworksonline.com/cda/content/applet/0,,NAV2-21_SAP204,00.shtml

Procedure:

1. With two sticks from the stick box, cross them perpendicularly.
2. Ask the children to estimate the size of each of the four angles formed. If they cannot guess, use a protractor or corner of a box. They should eventually see that all four angles are approximately 90 degrees. Ask them to note that two of the angles would add up to 180 degrees and would be a straight line.
3. Change the position of the sticks so that they cross at non-90 degree angles. Ask them to estimate the sizes of the four angles. Use a protractor if needed. Do this until someone observes that the opposite angles match.
4. Through a series of questions and observations, get the children to notice that no matter how you cross the sticks, the opposite angles will be the same. Also have them observe that if any of the two angles side-by-side are added, they will become a straight line, also known as supplementary angles.
5. Do a similar exercise, this time creating a 90 degree angle with two sticks and using another stick to divide the 90 degrees in a number of ways. Eventually, they should notice that the two angles added up equal 90 degrees. These are complementary angles.
6. Finally, place two sticks parallel and cross them with another stick at an angle. By measuring each angle, have them notice that opposite interior angles are always the same. Have them give their ideas why this is so.
7. In the same manner, have children observe the corresponding angles.
8. Lessons may be further enhanced with domino game, website, and/or software.



Lesson 7: Geometry Cabinet Nomenclature

This lesson is to familiarize students with the polygons and 2D shapes in the Montessori geometric cabinet.

Standards Addressed:

- Can identify circle, oval, ellipse, square, rectangle, triangle, quatrefoil, trapezoid, common quadrilateral, curvilinear triangle, rhombus, parallelogram
- Can create simple shapes using Draw application

Materials and Resources Needed:

- AutoShapes in Microsoft Word for children to re-create shapes
- Carnival Countdown: Pattern block roundup, Levels B, F
- Number Heroes: Geoboard, Levels C, K, P, or explore mode
- Cosmic Geometry: Geo Academy, Levels D, F
- Cosmic Geometry: Amazing Angles, Level E
- Montessori geometric cabinet
- Illuminations website:
<http://standards.nctm.org/document/eexamples/chap4/4.2/part2.htm>
- BrainPOP website: <http://www.BrainPOP.com/math/geometry/polygons>

Procedure:

1. One by one, take the shapes out of the drawers and allow students to give characteristics that they notice about the shapes. Encourage them to describe the shapes using terms they have learned, such as points, lines, and angles.
2. Do a three-part lesson to familiarize with the names of the shapes. (This is a circle. Can you find the circle? What shape is this?)
 - a. Circle = Latin "circus"
 - b. Oval = Latin "egg"
 - c. Ellipse = Greek "omission, defective"
 - d. Square = Latin "quadra = four"
 - e. Rectangle = Latin "upright angle"
 - f. Triangle = Latin "three angles"
 - g. Quatrefoil = Latin "four, leaf"
 - h. Trapezoid = Greek "table"
 - i. Common quadrilateral = Latin "four, side"
 - j. Curvilinear triangle = Latin "curve, lines, three, angles"
 - k. Rhombus = Greek "revolve, can be spun around"
 - l. Parallelogram = Greek "equivalent, side by side, written or drawn"
3. Students can trace shapes to make a booklet. Students can also take turns "hiding" a shape in a basket underneath a towel and have other students guess the shape by touch. Students can also play the "Mailman" game in which students sit in a circle with hands behind their backs. The "mailman" goes around placing a shape in each person's hands. Then each child has to guess the shape before looking at it.
4. Possible follow-up activities can include creating the polygons using the AutoShapes menu in Word or exploration with the software.
5. Teacher can test knowledge by having students create shapes on Number Heroes: Geoboard (explore mode) or on the Illuminations website. (i.e. "Create a blue hexagon, a red circle, and a green parallelogram.") Cosmic Geometry: Geo Academy also tests knowledge, though at a higher level.
6. Children may enjoy watching BrainPOP movie about polygons.

Lesson 8: Perimeter and Area

The purpose of this lesson is to have children explore ways to find the perimeter and area of a regular quadrilateral.

Standards Addressed:

- Can find perimeter and area of a simple square or rectangle

Materials and Resources Needed:

- Carnival Countdown: Pattern block roundup, Levels I, J, N, O, U (*has excellent perimeter ruler and area grid tools that children can use to solve the problems)
- Number Heroes: Geoboard, Levels G, O, W (Levels O and W have children creating shapes given a certain perimeter and area)
- Cosmic Geometry: Geo Academy
- Cosmic Geometry: Amazing Angles, Levels G, H (extremely challenging)
- Montessori stamp game
- Dry erase board

Procedure:

1. Create a small rectangle or square using the tiles from a stamp game. Turn the stamps so that the numbers do not show and explain that these are not being used this time as units, tens, etc., but as simple squares.
2. Tell the children to think of the shape you have made as a series of blocks in a city or as square feet in a yard or room. If the child had to walk around the shape, how many blocks or feet would he/she have to walk? Count around the shape you've created to see how many blocks.
3. Rearrange the tiles to create various rectangles and squares and repeat.
4. Once children get the idea of perimeter (Greek: peri = "around" meter = "measure") have them again look at a square or rectangle created with tiles. Have them imagine that it is a room that needs to be covered in carpet. How many square feet would be needed to cover the room? One can tell the answer by simply counting the squares.
5. Move to the dry erase board and draw a rectangle broken into even squares. On the sides, place the number of squares there are on each side. Have children find perimeter and area of the rectangle. Repeat until they begin to notice that to find the perimeter, one simply adds the length of the sides. To find the area, one multiplies the length of one side by the length of the other side.
6. Begin drawing rectangles and squares with only the numbers shown on two sides and no squares inside. Children should be able to make the transition.
7. Children can explore further with software, or test knowledge on Cosmic Geometry: Geo Academy.

Lesson 9: Symmetry

The purpose of this lesson is to give students experience with finding and creating lines of symmetry on a polygon.

Standards Addressed:

- Can identify a line of symmetry on a shape
- Can identify radial symmetry versus linear symmetry

Materials and Resources Needed:

- BrainPOP "Invertebrates" movie
- Carnival Countdown: Pattern block roundup, Levels G, R, V, X
- Number Heroes: Geoboard, Levels E, M, T
- Cosmic Geometry: Geo Academy
- Montessori geometric shapes
- String
- Blue constructive triangles

Procedure:

1. Using any regular polygon from the geometric cabinet, place a string across the shape that follows a line of symmetry. Ask children what they notice about how you placed the string. Do this with several polygons.
2. Repeat, but this time, do not place the string along a line of symmetry. Again ask children for feedback about how you placed your string.
3. Explain that a line of symmetry is a line that cuts a polygon into two equal polygons. If children would like more practice, have them trace a polygon, cut it out, then fold it along lines of symmetry.
4. Skills can be practiced on software.
5. Using the blue constructive triangles, create a design with radial symmetry. Ask children where the design repeats. Draw their attention to the imaginary lines of radial symmetry.
6. Have children watch BrainPOP's "Invertebrates" movie. Scientists often classify invertebrates based upon bilateral or radial symmetry.
7. Have children create other designs showing radial and linear symmetry.

Lesson 10: Types of Triangles

The purpose of this lesson is to give children experiences in classifying triangles according to sides and angles.

Standards Addressed:

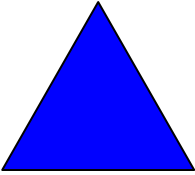
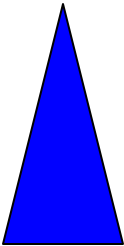
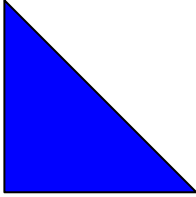

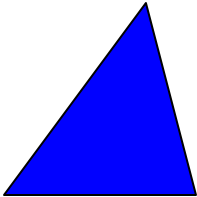
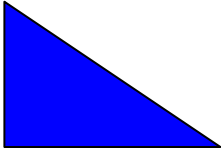
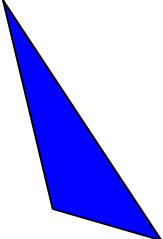
- Can classify a triangle according to angles and sides (right, acute, obtuse, equilateral, isosceles, scalene)

Materials and Resources Needed:

- Montessori geometric stick box
- Montessori geometric cabinet drawer with triangles
- red, blue, and yellow triangles from "Detective Adjective Game"
- Montessori constructive triangles (2nd box)
- Illuminations Geoboard:
<http://standards.nctm.org/document/eexamples/chap4/4.2/index.htm>
- BrainPOP website: "Types of Triangles"
<http://www.brainpop.com/math/geometry/typesoftriangles> or "Similar Triangles"
<http://www.brainpop.com/math/geometry/similartriangles>
- Number Heroes: Geoboard, Level Q or Exploration Mode
- Cosmic Geometry: Geo Academy
- Cosmic Geometry: Amazing Angles, Level C

Procedure:

1. Given each child three geometric sticks of the same size and two other sticks of two different sizes. As the sticks are color coded according to size, they should have three of one color and two more of two different colors.
2. Have children create a triangle with three of the same colored sticks. This triangle is an equilateral triangle because all three of its sides are the same.
3. Have children replace one of the sides of their equilateral triangle with a different colored stick. This will make an isosceles (Greek "with equal legs") triangle.
4. Repeat to create a scalene (Greek = "unequal") triangle.
5. See if children can identify the three types in the cabinet.
6. Using the sticks, have the children create triangles with all acute angles, one with a right angle, and one with an obtuse angle. See if they can identify acute, right, and obtuse angled triangles in the cabinet.
7. Begin asking children to create triangles with certain side and angle specifications (such as an obtuse-angled isosceles triangle.) Have them also try to create impossible triangles (right-angled or obtuse-angled equilateral triangles.) Children can create chart as seen on next page by tracing triangles from cabinet.
8. Children can play games as sited in Lesson 6.
9. Have children practice skills using constructive triangles (create an isosceles from two right scalene triangles), playing Detective Adjective game (partners; one is asked to find specific triangle based on size, color, sides, and angles), creating triangles on Illuminations website ("see if you can make a green right-angled triangle, a blue acute-angled isosceles triangle", etc.), or with software.
10. Children may enjoy seeing BrainPOP movies about triangles.

	Acute Angled	Right Angled	Obtuse Angled
Equilateral			
Isosceles			
Scalene			

Lesson 11: Traits of Triangles

The purpose of this lesson is for children to find that not all combinations of three sides will create a triangle and to find that the sum of the angles of a triangle will always be 180 degrees.

Standards Addressed:

- Can determine if three sides will create a triangle or not
- Can find missing angle of a triangle using the 180 rule

Materials and Resources Needed:

- Montessori triangle drawer
- Montessori geometric sticks
- protractor
- Cosmic Geometry: Geo Academy
- Cosmic Geometry: Amazing Angles, Levels J, K

Procedure:

1. Hand children one purple (5 cm), one green (15 cm), and one yellow stick (21 cm) from the geometric stick box and have them try to create a triangle.
2. Children will discover that it is impossible to create a triangle from these three sticks.
3. This is known as the Triangle Inequality Theorem. It states that the sum of two sides of a triangle must be greater than the third side. (Teachers: a great website explores this theorem: <http://www.mste.uiuc.edu/dildine/heron/triarea.html>)
4. Have children try to find other impossible combinations with the sticks.
5. Using the triangles from the geometric cabinet, give one triangle to each child and have the students measure the three angles with a protractor and add the angles. Once everyone is finished, go around and have them give their sums. Everyone's answer should be close to 180 degrees.
6. Use the applet from the Study Works website to explore this phenomenon.
7. If we know that all of the angles add up to 180 degrees, we can now find a missing angle if we know the value of the other two. Do several examples with the children so that they can practice this concept.
8. Further practice can be done on the software.

Lesson 12: Pythagorean Theorem

The purpose of this lesson is to expose third graders to the Pythagorean Theorem.

Standards Addressed:

- Can give definition of the Pythagorean Theorem and find a missing side of a triangle using the theorem

Materials and Resources Needed:

- Montessori Pythagorean material
- Cosmic Geometry: Geo Academy
- Cosmic Geometry: Amazing Angles
- Study Works website:
http://www.studyworksonline.com/cda/content/applet/0,,NAV2-21_SAP700,00.shtml
- Illuminations website:
<http://standards.nctm.org/document/eexamples/chap6/6.5/index.htm>
- BrainPOP website:
<http://www.brainpop.com/math/algebra/thepythagoreantheorem>

Procedure:

1. Place the triangle from the Pythagorean material on a rug. See if children can identify it as a right triangle. Draw their attention to the side that is opposite the right angle. This side is called the hypotenuse.
2. Explain that right triangles have special properties. If we create two squares with the same lengths of sides as the two sides of the right triangle that are not the hypotenuse, those two squares will fit into a square with the same length of sides as the hypotenuse.
3. Place the small squares from the Pythagorean material along side "a" and "b" sides of the triangle. Have volunteers come up and place the rest of the tiles to create the two squares. (Pythagoras: Greek philosopher, mathematician, religious reformer who lived from 582 – 500 BC.)
4. Have another volunteer group these tiles together underneath the hypotenuse. The children will see that they create a square that fits perfectly under the hypotenuse.
5. Together, work through the applets from the Study Works and Illuminations website to see the proofs of the Pythagorean Theorem. It helps to have these bookmarked on the computers so that children can revisit them. Sending the website addresses home for homework also allows for more experience with the concept.
6. Advanced practice can be done on the software. Children may enjoy watching BrainPOP movie about the Pythagorean Theorem. The movie has links to science concepts that are related to the Pythagorean Theorem.

Lesson 13: Regular Polygons

The purpose of this lesson is to familiarize students with nomenclature and attributes of common regular polygons.

Standards Addressed:

- Can identify and define a regular polygon
- Can create polygons with other shapes
- Can find the perimeter of a polygon
- Can find missing angle of a polygon based on the 360 rule

Materials and Resources Needed:

- Montessori geometric cabinet: pentagon, hexagon, heptagon, octagon, nonagon, and decagon (polygon = Greek "many sides," penta, hexa, etc. from Greek numbers)
- Montessori geometric sticks
- blue constructive triangles
- Montessori constructive triangles (1st box)
- Ruler
- protractor
- Illuminations websites
- Number Heroes: Geoboard, Levels C, K, G, O, W, or Exploration Mode
- Cosmic Geometry: Geo Academy (for advanced students)

Procedure:

1. With five various sizes of geometric sticks, create an irregular pentagon. Next, create a pentagon with equilateral sides, but not equal angles. Beside these, create a regular pentagon using five identical sticks. Ask children to note differences between the two shapes, particularly in relation to sides, angles, and lines of symmetry.
2. Explain that when shapes are created with equal sides and angles, they are called regular polygons. Those without equal sides or angles are called irregular polygons. Polygons get their names from the Greek words for the number of sides they have.
3. Take out the six shapes listed above from the cabinet and label each of them with a piece of paper or pre-created tags. Children can create a booklet by tracing the insets and labeling the polygons. Children can play games as sited in Lessons 7 and 10. Children can create and identify polygons using the software or on the Illuminations Geoboard:
<http://standards.nctm.org/document/eexamples/chap4/4.2/part2.htm>.
4. Using the constructive triangle boxes, have children experiment with creating certain polygons out of other shapes. Children can also work with tangrams. This activity is available online at the Illuminations website:
<http://standards.nctm.org/document/eexamples/chap4/4.4/index.htm>.
5. Show children that the sum of the length of the sides is known as the perimeter. Have children find the perimeters of the polygons from the cabinet using a ruler.

Lesson 14: Traits of the circle and ellipse

The purpose of this lesson is to familiarize children with traits and nomenclature associated with circles and ellipses.

Standards Addressed:

- Can identify circle and ellipse
- Can identify the foci
- Can find the radius, diameter, and circumference
- Can give a definition of pi

Materials and Resources Needed:

- Montessori geometric cabinet
- String and tacks
- flat cork board from geometric stick box
- Compasses
- BrainPOP website

Procedure:

1. Place a piece of paper on the cork board. Make a loop with the string. Secure one end around a pencil. Secure the other end to the center of the paper with a tack. Use the string to draw a circle, using the tack as the focus.
2. Ask children to estimate distance between the center of the circle and any point that you show on the perimeter of the circle. Do this several times until children notice that the answer is the same. See if they can come up with a definition of a circle based on this activity.
3. Place a clean sheet of paper on the board. With the loop of string, repeat step one, this time with the string looped around two tacks near the center of the board. The resulting shape should be an ellipse.
4. Based on this observation, see if children can come up with a definition of an ellipse. Ellipses have two foci, whereas circles have only one. It was once thought that the planets followed a circular orbit with the sun at the center, but now we know that the gravitational pull of other bodies in the sky cause the planets to follow an elliptical orbit.
5. Using the largest circle from the geometric cabinet, show children radius, diameter, and circumference (with a string.) Children should note that radius is $\frac{1}{2}$ the diameter.
6. With the string, show that the length of the string around the circle will go across the diameter of the circle a little more than three times. This relationship is known as pi.
7. Have children practice drawing circles with their compasses and labeling the diameter and radius. Older children can begin finding the circumference using $C = \pi D$. (radius = Latin "ray," diameter = Greek "passing through, measure," circumference = Latin "around, carry")
8. Students can further explore with BrainPOP's "Pi"
<http://www.brainpop.com/math/geometry/pi>

Lesson 15: Further study of circles

The purpose of this lesson is to expose third graders to the concepts of degrees in a circle, chords, arcs, and sectors.

Standards Addressed:

- Can find degree measurement of a sector and compare it to a circle being 360 degrees
- Can identify a chord, arc, and sector

Materials and Resources Needed:

- Montessori fraction cut-outs
- Montessori 360 degree template
- large circle from geometric cabinet
- Cosmic Geometry: Geo Academy, Level G
- Cosmic Geometry: Amazing Angles, Level F
- BrainPOP website

Procedure:

1. Place large circle inset into the 360 template. Explain that early mathematicians could have chosen any number to divide a circle, but that 360 has many, many factors. Have children list some of the factors of 360. Remind them of times they may have heard the term "turning a 360."
2. This makes it easier to divide a circle into sectors. Take the circle off of the template and begin placing ninths from the fraction cut-outs onto the template. Children can see that each sector is 40 degrees. See if children can come up with the degree equivalents of other sectors.
3. Draw the children's attention to the outside, circular edge of the fraction cut-outs. This curved line is known as an arc.
4. Finally, show a line going across the circle inset that does not intersect the center is known as a chord.
5. Watch the "Measuring Circles" animated movie on the BrainPOP website:
<http://www.BrainPOP.com/math/measurement/measuringcircles/index.weml>
6. Children can practice concepts using software.

Lesson 16: Similar, Equivalent, and Congruent

The purpose of this lesson is for children to be able to classify two shapes as being similar, equivalent, or congruent.

Standards Addressed:

- Can classify two shapes as being similar, equivalent, or congruent

Materials and Resources Needed:

- Montessori stamp game
- hexagon & triangle plates
- red, blue, and yellow triangles from Detective Adjective Game
- Number Heroes: Geoboard, Levels D, L, S
- Cosmic Geometry: Geo Academy, Level M
- Cosmic Geometry: Amazing Angles, Levels Q, R

Procedure:

1. Take two identical triangles, though different colors, from the Detective Adjective Game. Children should notice that they are exactly the same. These are called congruent (Latin = "come together") figures. One can be precisely placed on top of the other. The angles and sides are identical and in the same places.
2. Take out a larger or smaller version of the same triangle from the Game. Children should notice that it is exactly the same triangle, though a different size. Shapes that have identical proportions, though different sizes, are called similar shapes.
3. Create a rectangle using tiles from the stamp game. Have children give area as learned in lesson 7. Rearrange the tiles to create a different shape, but with the same amount of tiles. Ask children what the area of this shape is. (They should be the same.) Figures with different shapes, but equal areas are known as equivalent shapes.
4. With the hexagon and triangle plates, have children create shapes that are similar, congruent, and equivalent.
5. Children can further practice concepts with software.

Lesson 17: Montessori geometric solids

The purpose of this lesson is to familiarize students with the nomenclature and attributes of the ovoid, ellipsoid, cylinder, cube, sphere, square-based pyramid, triangular-based pyramid, triangular prism, rectangular prism, and cone.

Standards Addressed:

- Student can identify and define ovoid, ellipsoid, cylinder, cube, sphere, square-based pyramid, triangular-based pyramid, triangular prism, rectangular prism, cone
- Can find volume and surface area of simple solids
- Can create simple charts in a word processing document or in a spreadsheet

Materials and Resources Needed:

- Montessori geometric solids
- Montessori volume cube
- Units from Montessori number rods or cubing material
- Digital photos of each solid
- Zoo Zillions: 3D Gallery
- Calculating Crew: Dr. Gee's 3D Lab
- Cosmic Geometry: Amazing Angles, Levels M, N
- BrainPOP website

Procedure:

1. Do three-part lesson with children to familiarize them with the nomenclature of the solids. Have children note the 2D shapes that correspond to the 3D shapes. Games can be played as sited in lessons 7, 10, and 13 to help children learn names of shapes.
2. Based on earlier area lessons, have third graders find the area of each side of the cube and rectangular prism. Have them add up the areas of the sides. This is known as surface area.
3. Show third graders one cube from the volume cube. Ask them how many of these cubes they think will fit into the volume cube. Students can explore by lining up unit cubes along the sides of the volume cube. (Keep the volume cube closed so the answer isn't given away.) Once children have given guess, open up the volume cube and allow them to count how many actually fit. See if they can construct their own volume formula. ($V=L \times W \times h$) Volume is how much a 3D shape can hold.
4. Give third graders a general lesson of how to create a table in Word or Excel. Using the digital photos, have them create a chart of each solid. Column headings should include name of solid, 2D shapes on that can be identified within the solid, number of vertices, number of sides, number of edges. The photos of the solid should be inserted on the left-most column.
5. Further exploration can be done on the software.
6. Children may enjoy watching BrainPOP movies to enhance their understanding: Volume of Cylinders <http://www.brainpop.com/math/geometry/volumeofcylinders> and Volume of Prisms <http://www.brainpop.com/math/geometry/volumeofprisms>
7. cylinder = Greek "roller", prism = Greek "something sawed",

Lesson 18: Exploration with complex geometric solids

The purpose of this lesson is to give students exposure to working with geometric solids.

Standards Addressed:

- Student can identify characteristics of complex solids

Materials and Resources Needed:

- Straws
- Modeling clay
- Tape or glue
- Paper
- Scissors
- Illuminations website and grid to record information

Procedure:

1. Explain that there are more solids that children are going to work with besides only those included in the Montessori set.
2. Direct children to http://illuminations.nctm.org/index_d.aspx?id=406
3. Have them follow the "Student Sheets" link and have them print the packet.
4. Have children go through each of the activities.
5. These lessons will probably take several class periods to work through.

Lesson 19: Practical application of geometry

The purpose of this lesson is to give third grade students real-life experiences in using the concepts learned throughout the geometry unit.

Standards Addressed:

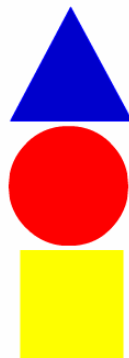
- Students will use geometry skills to serve practical purposes
- Students will create simple PowerPoint presentations

Materials and Resources Needed:

- WebQuest: "Designing and building a Montessori playground" available online at <http://ouray.cudenver.edu/~eaross/playground>
- Computers with internet access
- Popsicle sticks, glue, cardboard, aluminum foil, clay, sand, other materials for building
- PowerPoint
- Websites and booklet as provided by the WebQuest

Procedure:

1. Students will study the latest designs and research regarding playground design using the websites given in the WebQuest.
2. Based upon the final product of the WebQuest, students will build a small 3D model of their ideal playground.
3. Students will create a PowerPoint presentation to convince others of their ideas about the playground.



Designing and Building a Montessori Playground: A WebQuest for Third - Sixth Graders

by Elizabeth Ross Hubbell

The Montessori School of Denver needs a new playground and needs your help in designing it. Students want a fun playground with new equipment, but it needs to fit in the small space that we have. Also, the playground needs to be safe, nice-looking, comfortable, and within MSD's budget.

Click [here](#) to continue...

Home
Process
Safety Council
Building Material Council
Equipment Council
Budget Council
Share and Compare
Teacher Page
<small>Created on June 15, 2003 Last updated on July 11, 2003 Elizabeth Ross Hubbell</small>

Addendum

Montessori materials can be purchased from the websites listed below or created using found objects. Please note that many have specific dimensions.

www.bambini-montessori.com/index.htm

www.julianagroup.com/

www.nienhuis.com/

www.bruinsmontessori.com/

www.montessorimaterials.org/math.htm#geo

www.montessorimaterials.com/

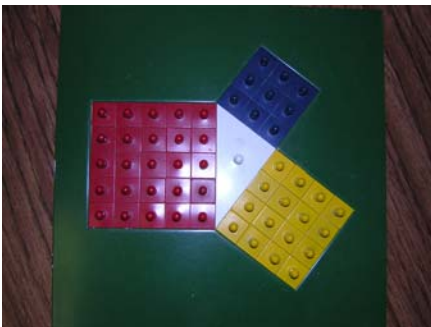
www.hellowood.com/mainp.htm



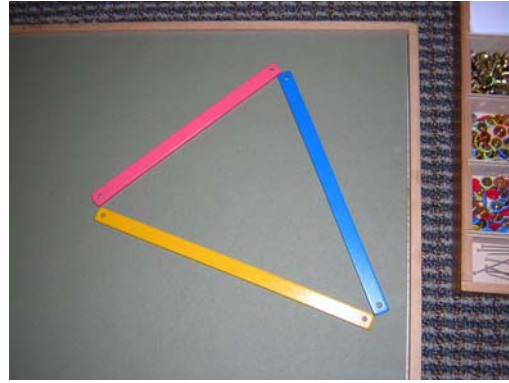
Geometric Solids



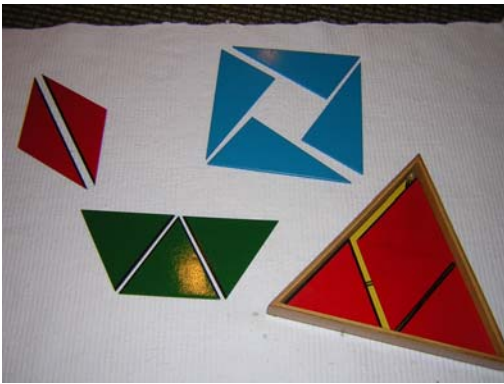
Geometric Cabinet



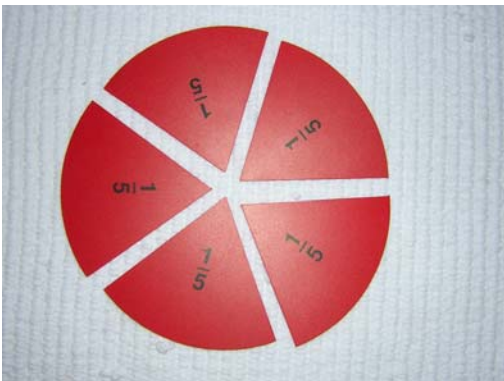
Pythagorean Material



Geometric Stick Box: sticks vary from 2 cm to 20 cm (even sizes only) and are color-coded by size. Tacks hold the sticks in place on the cork lid.



Geometric Triangle Boxes: various triangles that can be put together to create other shapes (i.e. two obtuse isosceles triangles can form a rhombus); closely resemble tangrams.



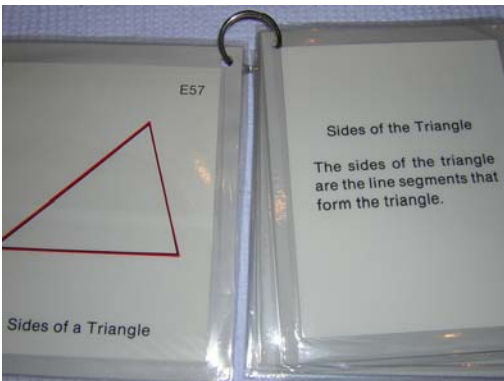
Fraction Cut-outs



Volume cube



360 degree template



Geometric nomenclature cards